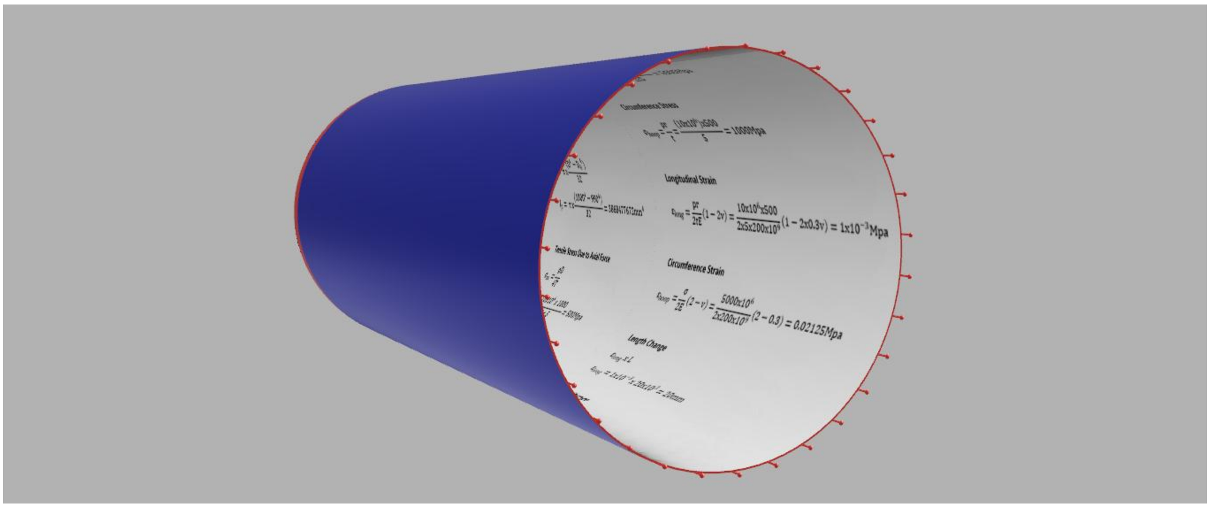
Cylinder Vessel



L = 20m D = 1000mm Thickness = 5mm

Youngs modulus = 200 Gpa Poissons ratio = 0.3

External pressure = 10 Mpa Axial torque = 1000kN

Longitudinal Stress

$$\sigma_{long} = -\frac{\rho r}{2t} = -\frac{(10x10^6)\ x\ 0.5}{2x0.005} = -500Mpa$$

Circumference Stress

$$\sigma_{hoop} = -\frac{\rho r}{t} = -\frac{(10x10^6)\ x\ 0.5}{0.005} = -1000Mpa$$

Longitudinal Strain

$$\varepsilon_{long} = -\frac{pr}{2tE}(1-2v) = -\frac{10x10^6\ x\ 0.5}{2x0.005x200x10^9}(1-2\ x\ 0.3) = -1x10^{-3}$$

Circumference Strain

$$\varepsilon_{hoop} = -\frac{pr}{tE}\left(1-\frac{v}{2}\right) = -\frac{10x10^6 x0.5}{0.005x200x10^9} x\left(1-\frac{v}{2}\right) = -4.25x10^{-3}$$

Length Change

ε_{long} x L

$\varepsilon_{long} = 1x10^{-3}$ x $20 = -20mm$

Diameter Change

ε_{hoop} x D

$\varepsilon_{hoop} = 4.25x10^{-3}$ x $1 = -4.25mm$

Principle Stress & Directions

Axial torque $= 1000kN$

$D = 1000mm$

Thickness $= 5mm$

Area

$D = 1000mm$

$D_0 = 990mm$

$$A = \pi \, x \frac{(D^2 - {D_0}^2)}{4}$$

$$A = \pi \, x \frac{(1000^2 - 990^2)}{4} = 15629.4mm^2$$

Polar Moment

$$I_p = \pi \, x \frac{(D^4 - {D_0}^4)}{32}$$

$$I_p = \pi \, x \frac{(1000^4 - 990^4)}{32} = 3868477672mm^4(pa)$$

Tensile Stress Due to Axial Force

$$\sigma_{xx} = -\frac{\rho D}{4t}$$

$$\sigma_{xx} = -\frac{10x10^6 \; x \; 1}{4 \; x \; 0.005} = -500Mpa$$

Axial Torque

$\mathrm{T} = 1000\mathrm{kN}$

Torsional Shear

$$\sigma_{xy} = \frac{TD}{2I_p}$$

$$\sigma_{xy} = \frac{1000x10^3 \text{ x } 1}{2x(3868477672x10^{-6})} = 129\mathrm{Mpa}$$

$$I_1 = \sigma_{ii} = \sigma_1 + \sigma_2 + \sigma_3 = \sigma_{xx}$$

$$I_2 = -\frac{1}{2}\left(\sigma_{ii}\,\sigma_{jj} - \sigma_{ij}\,\sigma_{ji}\right) = \frac{1}{2}\left(\sigma_{xy}\,\sigma_{yx} - \sigma_{yx}\,\sigma_{xy}\right) = \sigma_{xy}^2$$

$$I_3 = \det \sigma_{ij} = \sigma_{xx}\,\sigma_{yy}\,\sigma_{zz} - \sigma_{xx}\,\sigma_{yz}^2 - \sigma_{yy}\,\sigma_{xz}^2 - \sigma_{zz}\,\sigma_{xy}^2 + 2\sigma_{xy}\,\sigma_{yz}\,\sigma_{xz} = 0$$

So, we have

$$\sigma_\lambda^3 - I_1\,\sigma_\lambda^2 - I_2\,\sigma_\lambda - I_3 = \sigma_\lambda^3 - I_1\sigma_\lambda^2 - I_2\,\sigma_\lambda = \sigma_\lambda\,(\sigma_\lambda^2 - I_1\,\sigma_\lambda - I_2) = 0$$

Thus

$$\sigma_\lambda^2 - I_1\,\sigma_\lambda - I_2 = 0$$

Solution

$$\sigma_{1,2} = \frac{1}{2}I_1 \pm \frac{1}{2}\sqrt{I_1^2 + 4I_2^2}$$

Principle Stress

$$\sigma_1 = \frac{1}{2}\left(\sigma_{xx} + \sigma_{yy}\right) + \sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4\sigma_{xy}^2}$$

$$\sigma_1 = \frac{1}{2}\sigma_{xx} + \frac{1}{2}\sqrt{\sigma_{xx}^2 + 4\sigma_{xy}^2}$$

$$\sigma_1 = \frac{1}{2}\sigma_{xx} - \frac{1}{2}\sqrt{\sigma_{xx}^2 + 4\sigma_{xy}^2}$$

$$\sigma_{xx} = -500\text{Mpa}$$

$$\sigma_{xy} = +129\text{Mpa}$$

$$\sigma_1 = -\frac{1}{2}(500) - \left(\frac{1}{2}\right)\sqrt{(500^2) + (4\text{x}129^2)} = -531.3\text{Mpa}$$

$$\sigma_2 = \frac{1}{2}(500) + \left(\frac{1}{2}\right)\sqrt{(500^2) + (4\text{x}129^2)} = 31.3\text{Mpa}$$

$$\sigma_1 = -531.3\text{Mpa}$$

$$\sigma_2 = 31.3\text{Mpa}$$

$$\sigma_3 = 0$$

Normal Stress

$$\tau_{\text{normal}} = \frac{\sigma_1 + \sigma_2}{2}$$

$$\tau_{\text{normal}} = \frac{-531.3 + (31.3)}{2} = 250\text{Mpa}$$

Maximum Shear Stress

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{\max} = \frac{-531.03 - (31.3)}{2} = 281.3\text{Mpa}$$